# Visualizing Multiplication

## Lesson Topic

Area model for multiplication

## Grades

3–5

## Lesson Length

50 minutes

## NCTM Standards Addressed

- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes.
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes.
- Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.
- Build and draw geometric objects.
- Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute.
- Describe location and movement using common language and geometric vocabulary.
- Make and use coordinate systems to specify locations and to describe paths.
- Use geometric models to solve problems in other areas of mathematics, such as number and measurement.
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.

## Sample State Standards Addressed

- Select and use appropriate instruments and units for measuring quantities (e.g., perimeter, volume, area, weight, time, temperature).
- Develop formulas and procedures for determining measurements (e.g., area, volume, distance).
- Construct two- and three-dimensional shapes and figures using manipulatives, geoboards, and computer software.

## Student Objectives

Students will:

- represent multiplication of a two-digit number by a two-digit number as the area of a rectangle with dimensions of the two factors

- find patterns for the number of different base ten blocks in a rectangle representing a product
- use the patterns to find such products mentally.

## Grouping for Instruction

- Whole group for launch and closure
- Small groups for the investigation of transformations

## Overview of Lesson

Students use base ten blocks to represent whole numbers. For example, 123 can be represented by one flat (one hundred), two longs (two tens) and three units (three ones). Remind the students that one interpretation of multiplication is the area model. Show students how to measure a length and width of a rectangle that correspond to the two factors of a product of two two-digit numbers. They then determine which base ten blocks will fill in the corresponding rectangle. They use the values of the blocks to determine the product. Students record the results of several products done in this manner in a table and look for patterns. The students then use the patterns to find several products mentally.

## Background Information

Students should have used base ten blocks to represent numbers prior to this investigation and be familiar with the names for the different blocks. They should feel comfortable using a ruler to measure in centimeters. They should be able to draw perpendicular lines. Students need to have a solid understanding of place value up to hundreds.

## Materials and Equipment

- Overhead projector and transparencies
- Overhead base ten blocks
- A classroom set of base ten blocks
- Rulers
- Calculators

## Procedure

### A. Motivation and introduction

1. Tell students: "You are familiar with multiplying one-digit numbers such as five times six. Today we are going to look at multiplying a two-digit number by a two-digit number."
2. State: “Recall that the product of five and six can be interpreted as the area of a rectangle with dimensions five and six.” Illustrate this on the overhead projector.

3. Ask: “How could we use the area model to find the product (13)(34)?"
   (Take some students’ suggestions.)
4. Say: “If we represent 13 using base ten blocks it would be one long and three units. If you create a train with a long and 3 units, how long is the train?" Illustrate this on the overhead projector.
5. Ask: “How would you represent 34 using base ten blocks?” (Wait for responses.)
   “If you create a train with these base ten blocks, how long is the train?”
6. Put a transparency on the overhead that has a rectangle 13 centimeters by 34 centimeters. “The rectangle on this transparency is 13 centimeters by 34 centimeters. Notice that three flats fit in the area of this rectangle.” Ask:
   - “What is the value of three flats?” (Wait for responses.)
   - “How many longs will fit in the remaining area?”
   - “Can I trade in some longs for a flat? How many do I need?”
   - “What is the total value of the flats and longs?”
   - “How many units will fit in the remaining area?”
7. Ask: “What is the value of all the base ten blocks needed to cover the area of the rectangle? That is, what is the product of 13 and 34?”
8. Ask: “Do you think there are patterns when multiplying a two-digit number by a two-digit number? Let's find out.”

## B. Development (including discussion points and feedback)

1. Place the students in heterogeneous cooperative groups of about four students each.
2. Assign a task to each person in a team (leader, recorder, reporter, materials coordinator).
3. Ask the teams to complete the worksheet, “Multiplication Using Base Ten Blocks” (page 163).
4. Circulate among the groups, guiding them to complete the project and observing student interaction and understanding.
5. Have each team report to the class what they learned from the lesson.

## C. Summary and closure

1. Ask students to write two statements in their math journal about what they learned today.
2. Have several students share and build on other students’ responses.
   Guide students to observe the following:
   - used base ten blocks to show the multiplication of a two-digit number by a two-digit number
   - showed multiplication as the area of a rectangle
   - found patterns for determining the number of squares in a rectangle representing a product.

3. State: "Consider the product of 25 and 32. If you create a rectangle to represent this product, what dimensions should you use?"
    - "When you create this rectangle, how many flats will fit inside the rectangle? How do you know?"
    - "How many vertical longs will fit inside the remaining area of the rectangle formed? How do you know?"
    - "How many horizontal longs will fit inside the remaining area of the rectangle formed? How did you determine this?"
    - How many units will fit inside the remaining area of the rectangle formed? How can you find this number from the original product?
    - Can someone explain why these patterns hold?

### D. Assignment

Ask someone in your house to give you an example of a problem where you would have to multiply two two-digit numbers to solve the problem. Use what you learned in this lesson to find the product mentally. Check your answer with a calculator.

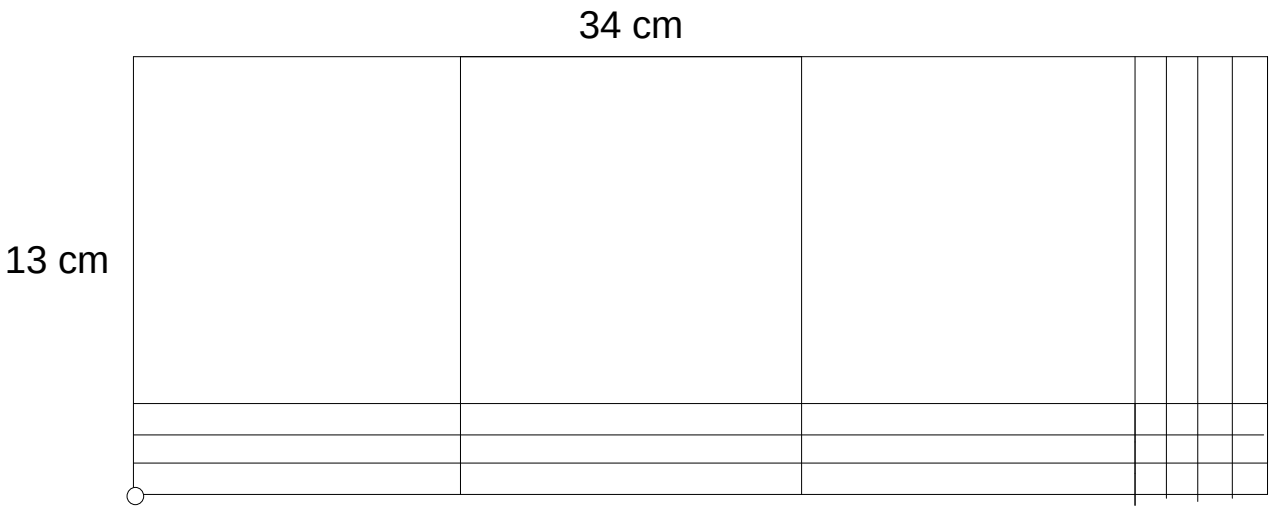
## Assessment

- Observe the students during the group work.
- Use a checklist to record any students who do not have the necessary background, so this can be rectified.
- Give each team a group grade on the project.
- Ask the students to explain in their mathematics journal what they learned during the lesson and any concepts that are still unclear.
- Ask students to explain how they could use the area of a rectangle to find the product of six and thirty-five.

## Worksheet:

# Multiplication Using Base Ten Blocks: Using Geometry to Visualize Multiplication

**Example:** We can represent the product of 13 and 34 as shown below. We can use trading to get the correct final answer. The procedure is as follows.




1. Measure 13 centimeters vertically. (The length of the rectangle equals the first factor.)
2. Measure 34 centimeters horizontally starting at the top of the vertical line. (The width of the rectangle equals the second factor.)
3. Use as few base ten blocks as possible to fill in the rectangle.
4. The sum of the values of these base ten blocks is the desired product.

1. Use this technique to find the following products. Fill in the following table for each product.

| Problem | # Flats | # Vertical Longs | # Horizontal Longs | # Units | Product |
|---|---|---|---|---|---|
| 13 x 34 | 3 | 4 | 9 | 12 | 442 |
| 12 x 23 | | | | | |
| 31 x 25 | | | | | |
| 16 x 13 | | | | | |

2. Refer to the table. How could you find the number of flats in the rectangle from the original problem?

3. Refer to the table. How could you find the number of vertical longs in the rectangle from the original problem?

4. Refer to the table. How could you find the number of horizontal longs in the rectangle from the original problem?

5. Refer to the table. How could you find the number of units in the rectangle from the original problem?

**Exercise:** Use the pattern you just discovered to find the following products mentally.

6. 31 x 22

7. 41 x 12

8. 52 x 45

9. 28 x 36

10. Why does this work?